

On Test of CPT Symmetry in CP -violating B Decays

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Abstract

Considering that the existing experimental limit for CPT violation is still poor, we explore various possible ways to test CPT symmetry in CP -violating B decays at e^+e^- B factories and hadron machines. We find that it is difficult to distinguish between the effect of direct CP violation and that of CPT violation in the time-integrated measurements of neutral B decays to CP eigenstates such as ψK_S and $\pi^+\pi^-$. Instead, a cleaner signal of small CPT violation may appear in the time-integrated CP asymmetries for a few non- CP -eigenstate channels, e.g., $B_d^0/\bar{B}_d^0 \rightarrow D^\pm\pi^\mp$ and $\bar{D}^{(*)0}K_S$. The time-dependent measurements of CP asymmetries are available, in principle, to limit the size of CPT -violating effects in neutral B -meson decays.

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I. Introduction

Tests of the fundamental symmetries and conservation laws have been an important topic in particle physics. Recently the necessity of testing CPT symmetry has been emphasized by several authors [1-3]. On the experimental side, the existing evidence for CPT invariance is rather poor. The limit for the strength of CPT -violating interaction in the $K^0 - \bar{K}^0$ system is about 10% of that of CP -violating interaction [2-4]. This means that CPT symmetry is tested only at the 10% level. On the theoretical side, the universality of CPT theorem is questionable since it is proved in local renormalizable field theory with a heavy use of the properties of asymptotic states [5]. This proof may not be applicable for QCD, where quarks and gluons are confined other than in a set of asymptotic states [3].

The simplest test of CPT invariance is to examine the equality of the masses and lifetimes of a particle and its antiparticle. Beyond the K -meson system, Kobayashi and Sanda are the first to suggest some ways to check CPT symmetry in B -meson decays at the future B factories [3]. With the assumption that the amplitudes for semileptonic decays satisfy the $\Delta Q = \Delta B$ rule and CPT invariance, they relaxed CPT symmetry for the mass matrix of the B_d mesons. They found that both $B_d^0 - \bar{B}_d^0$ mixing and CP asymmetries in neutral B decays will get modified if CPT violation is present. A key point of their work is that CP asymmetries in nonleptonic B decays such as $B_d^0/\bar{B}_d^0 \rightarrow \psi K_S$ may be more sensitive to the presence of small CPT -violating effects than the dileptonic decay rates of $B_d^0\bar{B}_d^0$ pairs.

In this work, we shall make an instructive analysis of the effects of CPT violation on CP -violating asymmetries in neutral B -meson decays. Both time-dependent and time-integrated CP asymmetries are calculated to meet various possible measurements at e^+e^- B factories and hadron machines. We suggest several ways for distinguishing CPT violation from direct CP violation in B decay amplitudes and indirect CP violation via interference between decay and mixing. We show that it is difficult to extract the CPT -violating information from the time-integrated measurements of neutral B decays to CP eigenstates such as $B_d^0/\bar{B}_d^0 \rightarrow \psi K_S$ and $\pi^+\pi^-$. Instead, cleaner signals of small CPT violation may appear in the time-integrated CP asymmetries of some non- CP -eigenstate channels, e.g., $B_d^0/\bar{B}_d^0 \rightarrow D^\pm\pi^\mp$ and $D^{(*)0}K_S$. Measurements of the time development of B_d^0 versus \bar{B}_d^0 decays are available, in principle, to limit the size of CPT -violating effects on CP asymmetries.

II. Decay probabilities

We begin with the mass eigenstates of the two B_d mesons [6]:

$$\begin{aligned} |B_1\rangle &= \frac{1}{\sqrt{|p_1|^2 + |q_1|^2}} \left(p_1 |B_d^0\rangle + q_1 |\bar{B}_d^0\rangle \right), \\ |B_2\rangle &= \frac{1}{\sqrt{|p_2|^2 + |q_2|^2}} \left(p_2 |B_d^0\rangle - q_2 |\bar{B}_d^0\rangle \right), \end{aligned} \quad (1)$$

where $p_{1,2}$ and $q_{1,2}$ are parameters of the $B_d^0 \bar{B}_d^0$ mass matrix elements. For convenience, the ratios of $q_{1,2}$ to $p_{1,2}$ can be further written as

$$\frac{q_1}{p_1} = e^{i\phi} \tan \frac{\theta}{2}, \quad \frac{q_2}{p_2} = e^{i\phi} \cot \frac{\theta}{2}, \quad (2)$$

where θ and ϕ are generally complex [6]. CPT symmetry requires $q_1/p_1 = q_2/p_2 = e^{i\phi}$ or $\mathcal{S} \equiv \cot \theta = 0$, and CP invariance requires that both $\mathcal{S} = 0$ and $\phi = 0$ hold. Furthermore, the proper-time evolution of an initially ($t = 0$) pure B_d^0 or \bar{B}_d^0 is given by

$$\begin{aligned} |B_d^0(t)\rangle &= e^{-(im + \frac{\Gamma}{2})t} \left[g_+(t) |B_d^0\rangle + \tilde{g}_+(t) |\bar{B}_d^0\rangle \right], \\ |\bar{B}_d^0(t)\rangle &= e^{-(im + \frac{\Gamma}{2})t} \left[\tilde{g}_-(t) |B_d^0\rangle + g_-(t) |\bar{B}_d^0\rangle \right], \end{aligned} \quad (3)$$

where

$$\begin{aligned} g_{\pm}(t) &= \cos^2 \frac{\theta}{2} e^{\pm(i\Delta m - \frac{\Delta\Gamma}{2})\frac{t}{2}} + \sin^2 \frac{\theta}{2} e^{\mp(i\Delta m - \frac{\Delta\Gamma}{2})\frac{t}{2}}, \\ \tilde{g}_{\pm}(t) &= \sin \frac{\theta}{2} \cos \frac{\theta}{2} \left[e^{(i\Delta m - \frac{\Delta\Gamma}{2})\frac{t}{2}} - e^{-(i\Delta m - \frac{\Delta\Gamma}{2})\frac{t}{2}} \right] e^{\pm i\phi}. \end{aligned} \quad (4)$$

In Eqs.(3) and (4) we have defined $m \equiv (m_1 + m_2)/2$, $\Gamma \equiv (\Gamma_1 + \Gamma_2)/2$, $\Delta m \equiv m_2 - m_1$, and $\Delta\Gamma \equiv \Gamma_1 - \Gamma_2$, where $m_{1,2}$ and $\Gamma_{1,2}$ are the mass and width of $B_{1,2}$.

Concentrating only on checking CPT symmetry in the mass matrix of the B_d mesons, here we assume that semileptonic B decays satisfy the $\Delta Q = \Delta B$ rule and CPT invariance. We also neglect CPT violation in the transition amplitudes for nonleptonic B decays. Relaxing these limitations can certainly be done, but the results will become too complicated. To obtain simple and instructive results, we further assume that

$$\frac{\Delta\Gamma}{\Gamma} = 0, \quad \text{Im}\phi = 0, \quad \text{Im}\theta = 0, \quad (5)$$

and $\mathcal{S} = \cot \theta$ is small and real. As examined in Ref.[3], large $\text{Im}\phi$ and $|\mathcal{S}|$ should be measured in the dileptonic decay ratios of $B_d^0 \bar{B}_d^0$ pairs at the $\Upsilon(4S)$. On the other hand, a

mode-independent analysis shows that $\Delta\Gamma/\Gamma$ is of the order $O(\leq 10^{-2})$ and has little effect on the time-integrated CP asymmetries in B_d decays [7]. In this work, we proceed with the above approximations to calculate the decay probabilities of B_d^0 and \bar{B}_d^0 mesons and explore the CPT -violating effects on CP asymmetries instructively. The more precise results will be presented elsewhere.

In view of current experiments on B -meson physics and proposals for future B factories [8], we consider two categories of experimental environments for the production and decays of neutral B mesons.

A. Decays of incoherent B_d^0 and \bar{B}_d^0 mesons

In a hadronic production environment [9] or in high energy e^+e^- reactions (e.g., at the Z peak), B_d^0 and \bar{B}_d^0 mesons are produced incoherently. The identification of the flavor of a neutral B meson can make use of hadrons produced nearby in phase space [10]. In this case, the time-dependent probability for B_d^0 (\bar{B}_d^0) decaying into a hadronic state $|f\rangle$ is given by

$$\begin{aligned}\Pr[B_d^0(t) \rightarrow f] &\propto |A_f|^2 e^{-\Gamma t} |g_+(t) + \tilde{g}_+(t)\zeta_f|^2, \\ \Pr[\bar{B}_d^0(t) \rightarrow f] &\propto |A_f|^2 e^{-\Gamma t} |\tilde{g}_-(t) + g_-(t)\zeta_f|^2,\end{aligned}\quad (6)$$

where

$$A_f \equiv \langle f|H|B_d^0 \rangle, \quad \bar{A}_f \equiv \langle f|H|\bar{B}_d^0 \rangle, \quad \zeta_f \equiv \frac{\bar{A}_f}{A_f}. \quad (7)$$

With the approximations made in Eq.(5), we simplify Eq.(6) as

$$\begin{aligned}\Pr[B_d^0(t) \rightarrow f] &\propto |A_f|^2 e^{-\Gamma t} \left\{ (1 + |\xi_f|^2) \overset{(-)}{+} (1 - |\xi_f|^2) \cos(\Delta mt) \right. \\ &\quad \left. \overset{(+)}{-} 2\text{Im}\xi_f \sin(\Delta mt) \overset{(-)}{+} 2\mathcal{S}\text{Re}\xi_f [1 - \cos(\Delta mt)] \right\},\end{aligned}\quad (8)$$

where $\xi_f \equiv e^{i\phi}\zeta_f$, $|\xi_f| = |\zeta_f|$, and those $O(\mathcal{S}^2)$ terms have been neglected. Integrating $\Pr[B_d^0(t) \rightarrow f]$ over t , we obtain the time-independent decay probabilities:

$$\Pr(B_d^0 \rightarrow f) \propto |A_f|^2 \left[\frac{1 + |\xi_f|^2}{2} \overset{(-)}{+} \frac{1}{1 + x_d^2} \frac{1 - |\xi_f|^2}{2} \overset{(+)}{-} \frac{x_d}{1 + x_d^2} \text{Im}\xi_f \overset{(-)}{+} \frac{x_d^2}{1 + x_d^2} \mathcal{S}\text{Re}\xi_f \right], \quad (9)$$

where $x_d \equiv \Delta m/\Gamma$ is a measurable for $B_d^0 - \bar{B}_d^0$ mixing. Different from the previous results in the literature, here a linear \mathcal{S} term, which implies CPT violation in the $B_d^0 \bar{B}_d^0$ mass

matrix, appears in the decay probabilities. Note that in Ref.[3] $|\zeta_f| = 1$ has been taken for $B_d^0/\bar{B}_d^0 \rightarrow \psi K_S$. Such an approximation is generally inappropriate (see Sect.III.A), since significant direct CP violation may exist in decay amplitudes for many neutral B channels.

For the case that B_d^0 (\bar{B}_d^0) decays into $|\bar{f}\rangle$, the CP -conjugate state of $|f\rangle$, the corresponding decay probability $\text{Pr}[B_d^0(t) \rightarrow \bar{f}]$ ($\text{Pr}[\bar{B}_d^0(t) \rightarrow \bar{f}]$) can be obtained from $\text{Pr}[\bar{B}_d^0(t) \rightarrow f]$ ($\text{Pr}[B_d^0(t) \rightarrow f]$) by the replacements $A_f \rightarrow \bar{A}_{\bar{f}}$, $\zeta_f \rightarrow \bar{\zeta}_{\bar{f}}$, and $\xi_f \rightarrow \bar{\xi}_{\bar{f}}$, where

$$\bar{A}_{\bar{f}} \equiv \langle \bar{f} | H | \bar{B}_d^0 \rangle, \quad A_{\bar{f}} \equiv \langle \bar{f} | H | B_d^0 \rangle, \quad \bar{\zeta}_{\bar{f}} \equiv \frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}}, \quad \bar{\xi}_{\bar{f}} \equiv e^{-i\phi} \bar{\zeta}_{\bar{f}}. \quad (10)$$

With the same replacements, one can obtain the time-integrated decay probabilities $\text{Pr}(B_d^0 \xrightarrow{(-)} \bar{f})$ from Eq.(9).

B. Decays of coherent $B_d^0 \bar{B}_d^0$ pairs

At the $\Upsilon(4S)$ resonance, the B 's are produced in a two-body ($B_u^+ B_u^-$ and $B_d^0 \bar{B}_d^0$) state with definite charge parity. The two neutral B mesons mix coherently until one of them decays. Thus one can use the semileptonic decays of one meson to tag the flavor of the other meson decaying into a flavor-nonspecific hadronic final state. The unique experimental advantages of studying b -quark physics in this energy region are well known [11], and both symmetric and asymmetric $e^+e^- B$ factories will be built in the near future on the basis of such threshold collisions [12].

The time-dependent wave function for a $B_d^0 \bar{B}_d^0$ pair produced at the $\Upsilon(4S)$ can be written as

$$\frac{1}{\sqrt{2}} \left[|B_d^0(\vec{k}, t)\rangle \otimes |\bar{B}_d^0(-\vec{k}, t)\rangle + C |B_d^0(-\vec{k}, t)\rangle \otimes |\bar{B}_d^0(\vec{k}, t)\rangle \right], \quad (11)$$

where \vec{k} is the three-momentum vector of the B_d mesons, and $C = \pm$ is the charge parity of the $B_d^0 \bar{B}_d^0$ pair. Supposing one B_d meson decaying into a semileptonic state $|l^\pm X^\mp\rangle$ at (proper) time t_1 and the other into a nonleptonic state $|f\rangle$ at time t_2 , the time-dependent probabilities for such joint decays are given by

$$\begin{aligned} \text{Pr}(l^+ X^-, t_1; f, t_2)_C &\propto |A_l|^2 |A_f|^2 e^{-\Gamma(t_1+t_2)} |g_+(t_1)[\tilde{g}_-(t_2) + \zeta_f g_-(t_2)] \\ &\quad + C \tilde{g}_-(t_1)[g_+(t_2) + \zeta_f \tilde{g}_+(t_2)]|^2, \end{aligned}$$

$$\begin{aligned} \Pr(l^- X^+, t_1; f, t_2)_C &\propto |A_l|^2 |A_f|^2 e^{-\Gamma(t_1+t_2)} [\tilde{g}_+(t_1)[\tilde{g}_-(t_2) + \zeta_f g_-(t_2)] \\ &\quad + C g_-(t_1)[g_+(t_2) + \zeta_f \tilde{g}_+(t_2)]^2, \end{aligned} \quad (12)$$

where $A_l \equiv \langle l^+ X^- | H | B_d^0 \rangle \stackrel{CPT}{=} \langle l^- X^+ | H | \bar{B}_d^0 \rangle$. By applying Eq.(5), Eq.(6) is further simplified as

$$\begin{aligned} \Pr(l^\pm X^\mp, t_1; f, t_2)_C &\propto |A_l|^2 |A_f|^2 e^{-\Gamma(t_1+t_2)} \left\{ (1 + |\xi_f|^2) \mp (1 - |\xi_f|^2) \cos[\Delta m(t_2 + Ct_1)] \right. \\ &\quad \left. \pm 2\text{Im}\xi_f \sin[\Delta m(t_2 + Ct_1)] \right. \\ &\quad \left. \pm 2\mathcal{S}\text{Re}\xi_f [C - (1 + C) \cos(\Delta mt) + \cos[\Delta m(t_2 + Ct_1)]] \right\}, \end{aligned} \quad (13)$$

Similar to Eq.(8), here a CPT -violating term proportional to \mathcal{S} appears.

Within limits of our present detector technology, we have to consider the feasibility for an e^+e^- collider to measure the time development of the decay probabilities and CP asymmetries. For a symmetric collider running at the $\Upsilon(4S)$ resonance, the mean decay length of B 's is insufficient for the measurement of $(t_2 - t_1)$ [11]. On the other hand, the quantity $(t_2 + t_1)$ cannot be measured in a symmetric or asymmetric storage ring operating at the $\Upsilon(4S)$, unless the bunch lengths are much shorter than the decay lengths [11,12]. Therefore, only the time-integrated measurements are available at a symmetric B factory. Integrating $\Pr(l^\pm X^\mp, t_1; f, t_2)_C$ over t_1 and t_2 , we obtain

$$\Pr(l^\pm X^\mp, f)_- \propto |A_l|^2 |A_f|^2 \left[\frac{1 + |\xi_f|^2}{2} \mp \frac{1}{1 + x_d^2} \frac{1 - |\xi_f|^2}{2} \mp \frac{x_d^2}{1 + x_d^2} \mathcal{S}\text{Re}\xi_f \right], \quad (14)$$

and

$$\begin{aligned} \Pr(l^\pm X^\mp, f)_+ &\propto |A_l|^2 |A_f|^2 \left[\frac{1 + |\xi_f|^2}{2} \mp \frac{1 - x_d^2}{(1 + x_d^2)^2} \frac{1 - |\xi_f|^2}{2} \pm \frac{2x_d}{(1 + x_d^2)^2} \text{Im}\xi_f \right. \\ &\quad \left. \mp \frac{x_d^2(1 - x_d^2)}{(1 + x_d^2)^2} \mathcal{S}\text{Re}\xi_f \right]. \end{aligned} \quad (15)$$

For an asymmetric collider running in this energy region, one might want to integrate Eq.(13) only over $(t_2 + t_1)$ in order to measure the development of the decay probabilities with $\Delta t \equiv (t_2 - t_1)$ [11]. In this case, we obtain

$$\begin{aligned} \Pr(l^\pm X^\mp, f; \Delta t)_- &\propto |A_l|^2 |A_f|^2 e^{-\Gamma|\Delta t|} \left\{ \frac{1 + |\xi_f|^2}{2} \mp \frac{1 - |\xi_f|^2}{2} \cos(\Delta m \Delta t) \right. \\ &\quad \left. \pm \text{Im}\xi_f \sin(\Delta m \Delta t) \mp \mathcal{S}\text{Re}\xi_f [1 - \cos(\Delta m \Delta t)] \right\}, \end{aligned} \quad (16)$$

and

$$\begin{aligned}
\Pr(l^\pm X^\mp, f; \Delta t)_+ &\propto |A_l|^2 |A_f|^2 e^{-\Gamma|\Delta t|} \left\{ \frac{1 + |\xi_f|^2}{2} \mp \frac{1}{\sqrt{1 + x_d^2}} \frac{1 - |\xi_f|^2}{2} \cos(\Delta m \Delta t + \phi_{x_d}) \right. \\
&\quad \left. \pm \frac{1}{\sqrt{1 + x_d^2}} \text{Im} \xi_f \sin(\Delta m \Delta t + \phi_{x_d}) \right. \\
&\quad \left. \mp \mathcal{S} \text{Re} \xi_f \left[\frac{4 - x_d^2}{4 + x_d^2} - \frac{1}{\sqrt{1 + x_d^2}} \cos(\Delta m \Delta t + \phi_{x_d}) \right] \right\}, \tag{17}
\end{aligned}$$

where $\phi_{x_d} \equiv \tan^{-1} x_d$.

For the case that one neutral B meson decays into $|l^\mp X^\pm\rangle$ at time t_1 and the other decays into $|\bar{f}\rangle$ (the CP -conjugate state of $|f\rangle$) at time t_2 , the corresponding decay probabilities $\Pr(l^\mp X^\pm, t_1; \bar{f}, t_2)_C$ can be obtained from Eq.(13) by the replacements in Eq.(10). In a similar manner, one can obtain $\Pr(l^\mp X^\pm, \bar{f})_\pm$ and $\Pr(l^\mp X^\pm, \bar{f}; \Delta t)_\pm$ straightforwardly from Eqs.(14-17).

III. CP asymmetries and CPT -violating effects

The difference between the decay probabilities associated with $B_d^0 \rightarrow f$ and $\bar{B}_d^0 \rightarrow \bar{f}$ is a basic signal for CP violation. For the decays of incoherent B_d^0 and \bar{B}_d^0 mesons, the time-dependent and time-integrated CP -violating asymmetries are defined by

$$\mathcal{A}(t) \equiv \frac{\Pr[B_d^0(t) \rightarrow f] - \Pr[\bar{B}_d^0(t) \rightarrow \bar{f}]}{\Pr[B_d^0(t) \rightarrow f] + \Pr[\bar{B}_d^0(t) \rightarrow \bar{f}]} \tag{18}$$

and

$$\mathcal{A} \equiv \frac{\Pr(B_d^0 \rightarrow f) - \Pr(\bar{B}_d^0 \rightarrow \bar{f})}{\Pr(B_d^0 \rightarrow f) + \Pr(\bar{B}_d^0 \rightarrow \bar{f})}, \tag{19}$$

respectively. Corresponding to the possible measurements for joint $B_d^0 \bar{B}_d^0$ decays at symmetric (S) and asymmetric (A) e^+e^- B factories, we define the CP -violating asymmetries as

$$\mathcal{A}_C^S \equiv \frac{\Pr(l^- X^+, f)_C - \Pr(l^+ X^-, \bar{f})_C}{\Pr(l^- X^+, f)_C + \Pr(l^+ X^-, \bar{f})_C}, \tag{20}$$

and

$$\mathcal{A}_C^A(\Delta t) \equiv \frac{\Pr(l^- X^+, f; \Delta t)_C - \Pr(l^+ X^-, \bar{f}; \Delta t)_C}{\Pr(l^- X^+, f; \Delta t)_C + \Pr(l^+ X^-, \bar{f}; \Delta t)_C}. \tag{21}$$

In the following, we calculate these asymmetries for two categories of neutral B decays and discuss the small CPT -violating effects on them.

A. CP -eigenstate decays

We first consider the B_d^0 and \bar{B}_d^0 decays to CP eigenstates (i.e., $|\bar{f}\rangle = \pm|f\rangle$) such as ψK_S , $\pi^+\pi^-$, and $\pi^0 K_S$. With the phase convention $CP|B_d^0\rangle = |\bar{B}_d^0\rangle$ and the approximations in Eq.(5), we have $A_{\bar{f}} = \pm A_f$, $\bar{A}_{\bar{f}} = \pm \bar{A}_f$, $\bar{\zeta}_{\bar{f}} = 1/\zeta_f$, and $\bar{\xi}_{\bar{f}} = 1/\xi_f$. For convenience, we define three characteristic quantities:

$$\mathcal{U} = \frac{1 - |\xi_f|^2}{1 + |\xi_f|^2}, \quad \mathcal{V} = \frac{-2\text{Im}\xi_f}{1 + |\xi_f|^2}, \quad \mathcal{W} = \frac{2\text{Re}\xi_f}{1 + |\xi_f|^2}. \quad (22)$$

Nonvanishing \mathcal{U} and \mathcal{V} imply the CP violation in the decay amplitude and the one from interference between decay and mixing [13], respectively. \mathcal{W} (proportional to \mathcal{S}) is a measure of CPT violation in the mass matrix of the neutral B mesons.

For the decays of incoherent B_d^0 and \bar{B}_d^0 mesons into CP eigenstates, we obtain the CP asymmetries:

$$\mathcal{A}(t) = \mathcal{U}\cos(\Delta mt) + \mathcal{V}\sin(\Delta mt) + \mathcal{W}[1 - \cos(\Delta mt)], \quad (23)$$

and

$$\mathcal{A} = \frac{1}{1 + x_d^2}\mathcal{U} + \frac{x_d}{1 + x_d^2}\mathcal{V} + \frac{x_d^2}{1 + x_d^2}\mathcal{W}. \quad (24)$$

For symmetric and asymmetric e^+e^- collisions at the $\Upsilon(4S)$, the corresponding CP asymmetries in $(B_d^0\bar{B}_d^0)_C \rightarrow (l^\pm X^\mp)f$ are given by

$$\begin{aligned} \mathcal{A}_-^S &= \frac{1}{1 + x_d^2}\mathcal{U} + \frac{x_d^2}{1 + x_d^2}\mathcal{W}, \\ \mathcal{A}_+^S &= \frac{1 - x_d^2}{(1 + x_d^2)^2}\mathcal{U} + \frac{2x_d}{(1 + x_d^2)^2}\mathcal{V} + \frac{x_d^2(1 - x_d^2)}{(1 + x_d^2)^2}\mathcal{W}; \end{aligned} \quad (25)$$

and

$$\begin{aligned} \mathcal{A}_-^A(\Delta t) &= \mathcal{U}\cos(\Delta m\Delta t) + \mathcal{V}\sin(\Delta m\Delta t) + \mathcal{W}[1 - \cos(\Delta m\Delta t)], \\ \mathcal{A}_+^A(\Delta t) &= \frac{1}{\sqrt{1 + x_d^2}} [\mathcal{U}\cos(\Delta m\Delta t + \phi_{x_d}) + \mathcal{V}\sin(\Delta m\Delta t + \phi_{x_d})] \\ &\quad + \mathcal{W} \left[\frac{4 - x_d^2}{4 + x_d^2} - \frac{1}{\sqrt{1 + x_d^2}} \cos(\Delta m\Delta t + \phi_{x_d}) \right]. \end{aligned} \quad (26)$$

From the above equations we observe that all the CP asymmetries get modified if CPT violation is present. Two remarks are in order.

(1) \mathcal{A}_-^S is neither a pure measure of direct CP violation in the decay amplitude (i.e., $|\xi_f| \neq 1$) nor that of small CPT violation in the mass matrix, but a combination of both of them. If \mathcal{S} happens to take the following special value:

$$\mathcal{S} = \frac{1}{2x_d^2} \frac{|\zeta_f|^2 - 1}{\text{Re}\xi_f}, \quad (27)$$

the effects of CP and CPT violation will completely cancel out in \mathcal{A}_-^S . In \mathcal{A} and \mathcal{A}_+^S , the CP asymmetry from the interference between decay and mixing is commonly dominant. A combination of measurements of \mathcal{A}_-^S and \mathcal{A} (or \mathcal{A}_+^S) can in principle determine \mathcal{V} or $\text{Im}\xi_f$ unambiguously, as the relations

$$\mathcal{V} = \frac{1+x_d^2}{x_d} (\mathcal{A} - \mathcal{A}_-^S) = \frac{(1+x_d^2)^2}{2x_d} \left[\mathcal{A}_+^S - \frac{1-x_d^2}{1+x_d^2} \mathcal{A}_-^S \right] \quad (28)$$

hold.

(2) CPT -violating effects can be probed by measuring the time development of the CP asymmetries at the $\Upsilon(4S)$ or at a hadronic B factory. With the help of

$$\mathcal{A}_+^A(\Delta t) = \frac{1}{\sqrt{1+x_d^2}} \mathcal{A}_-^A \left(\Delta t + \frac{\phi_{x_d}}{\Delta m} \right) + \left(\frac{4-x_d^2}{4+x_d^2} - \frac{1}{\sqrt{1+x_d^2}} \right) \mathcal{W}, \quad (29)$$

the magnitude of \mathcal{W} or \mathcal{S} may be definitely limited by comparing the data on $\mathcal{A}_+^A(\Delta t)$ and $\mathcal{A}_-^A(\Delta t)$. In addition, nonvanishing signals of direct CP violation and CPT violation can appear on some special points of $\mathcal{A}(t)$ and $\mathcal{A}_\pm^A(\Delta t)$. For example,

$$\mathcal{U} = \mathcal{A} \left(\frac{2n\pi}{\Delta m} \right) = \mathcal{A}_-^A \left(\frac{2n\pi}{\Delta m} \right), \quad (30)$$

and

$$\begin{aligned} 2\mathcal{W} &= \mathcal{A} \left(\frac{2n\pi}{\Delta m} \right) + \mathcal{A} \left(\frac{(2n+1)\pi}{\Delta m} \right) \\ &= \mathcal{A}_-^A \left(\frac{2n\pi}{\Delta m} \right) + \mathcal{A}_-^A \left(\frac{(2n+1)\pi}{\Delta m} \right) \\ &= \frac{4+x_d^2}{4-x_d^2} \left[\mathcal{A}_+^A \left(\frac{(2n+\frac{1}{2})\pi}{\Delta m} - \frac{\phi_{x_d}}{\Delta m} \right) + \mathcal{A}_+^A \left(\frac{(2n-\frac{1}{2})\pi}{\Delta m} - \frac{\phi_{x_d}}{\Delta m} \right) \right], \end{aligned} \quad (31)$$

where $n = 0, \pm 1, \pm 2$, and so on.

It is worthwhile at this point to emphasize that measuring the time-integrated asymmetry \mathcal{A}_-^S for B_d^0 versus $\bar{B}_d^0 \rightarrow \psi K_S$ is not so good to probe CPT violation in the B_d mass matrix. As discussed in Ref.[14], the so-called hairpin channel may contribute to these two decay

modes. Thus a small deviation of $|\xi_{\psi K_S}|$ from unity is possible. With the help of two-loop effective weak hamiltonians and factorization approximations, the ratio of hairpin to tree-level amplitudes is estimated to be as large as 8% [14]. Therefore, direct CP violation should be taken into account when we study the fine effect of CPT violation in $B_d^0/\bar{B}_d^0 \rightarrow \psi K_S$ and other neutral B decays.

B. Non- CP -eigenstate decays

Now we consider the case that B_d^0 and \bar{B}_d^0 decay to a common non- CP eigenstate (i.e., $|\bar{f}\rangle \neq \pm|f\rangle$) but their amplitudes A_f ($A_{\bar{f}}$) and $\bar{A}_{\bar{f}}$ (\bar{A}_f) contain only a single weak phase. Most of such decays occur through the quark transitions $\bar{b} \rightarrow u\bar{c} \begin{smallmatrix} (-) \\ q \end{smallmatrix}$ and $c\bar{u} \begin{smallmatrix} (-) \\ q \end{smallmatrix}$ (with $q = d, s$), and typical examples are $B_d^0/\bar{B}_d^0 \rightarrow D^\pm \pi^\mp$ and $\bar{D}^{(*)0} K_S$. In this case, no measurable direct CP violation arises in the decay amplitudes since $|\bar{A}_{\bar{f}}| = |A_f|$, $|\bar{A}_f| = |A_{\bar{f}}|$, $|\bar{\zeta}_{\bar{f}}| = |\zeta_f|$, and $|\bar{\xi}_{\bar{f}}| = |\xi_f|$ [15]. For convenience, we define

$$\tilde{\mathcal{U}} = \frac{1 - |\xi_f|^2}{1 + |\xi_f|^2}, \quad \tilde{\mathcal{V}}_\pm = \frac{-\text{Im}(\xi_f \pm \bar{\xi}_{\bar{f}})}{1 + |\xi_f|^2}, \quad \tilde{\mathcal{W}}_\pm = \frac{\mathcal{S}\text{Re}(\xi_f \pm \bar{\xi}_{\bar{f}})}{1 + |\xi_f|^2}. \quad (32)$$

Note that here a nonzero $\tilde{\mathcal{U}}$ does not mean CP violation in the decay amplitude. $\tilde{\mathcal{V}}_-$ and $\tilde{\mathcal{W}}_-$, which imply indirect CP violation and CPT violation, will contribute to the CP asymmetries in this kind of decay modes.

For the decays of incoherent B_d^0 and \bar{B}_d^0 mesons, we obtain the time-dependent CP asymmetry as follows:

$$\mathcal{A}(t) = \frac{\tilde{\mathcal{V}}_- \sin(\Delta mt) + \tilde{\mathcal{W}}_- [1 - \cos(\Delta mt)]}{1 + \tilde{F}(\Delta mt)}, \quad (33)$$

where \tilde{F} is a function defined as

$$\tilde{F}(y) \equiv \tilde{\mathcal{U}} \cos y + \tilde{\mathcal{V}}_+ \sin y + \tilde{\mathcal{W}}_+ (1 - \cos y). \quad (34)$$

In this case, the time-integrated CP asymmetry is given by

$$\mathcal{A} = \frac{x_d \tilde{\mathcal{V}}_- + x_d^2 \tilde{\mathcal{W}}_-}{1 + x_d^2 + \tilde{\mathcal{U}} + x_d \tilde{\mathcal{V}}_+ + x_d^2 \tilde{\mathcal{W}}_+}. \quad (35)$$

For symmetric and asymmetric e^+e^- collisions at the $\Upsilon(4S)$ resonance, the corresponding CP asymmetries in the decay modes in question are given as

$$\begin{aligned} \mathcal{A}_-^S &= \frac{x_d^2 \tilde{\mathcal{W}}_-}{1 + x_d^2 + \tilde{\mathcal{U}} + x_d^2 \tilde{\mathcal{W}}_+}, \\ \mathcal{A}_+^S &= \frac{2x_d \tilde{\mathcal{V}}_- + x_d^2 (1 - x_d^2) \tilde{\mathcal{W}}_-}{(1 + x_d^2)^2 + (1 - x_d^2) \tilde{\mathcal{U}} + 2x_d \tilde{\mathcal{V}}_+ + x_d^2 (1 - x_d^2) \tilde{\mathcal{W}}_+}; \end{aligned} \quad (36)$$

and

$$\begin{aligned}\mathcal{A}_-^A(\Delta t) &= \frac{\tilde{\mathcal{V}}_- \sin(\Delta m \Delta t) + \tilde{\mathcal{W}}_- [1 - \cos(\Delta m \Delta t)]}{1 + \tilde{F}(\Delta m \Delta t)}, \\ \mathcal{A}_+^A(\Delta t) &= \frac{\tilde{\mathcal{V}}_- \sin(\Delta m \Delta t + \phi_{x_d}) + \tilde{\mathcal{W}}_- \left[\frac{4-x_d^2}{4+x_d^2} \sqrt{1+x_d^2} - \cos(\Delta m \Delta t + \phi_{x_d}) \right]}{\sqrt{1+x_d^2} + \tilde{F}(\Delta m \Delta t + \phi_{x_d}) + \tilde{\mathcal{W}}_+ \left[\frac{4-x_d^2}{4+x_d^2} \sqrt{1+x_d^2} - 1 \right]}. \quad (37)\end{aligned}$$

It should be noted that, in contrast with the asymmetry \mathcal{A}_-^S in $B_d^0 \bar{B}_d^0$ decays into CP eigenstates (see Eq.(25)), here \mathcal{A}_-^S is a pure measure of CPT violation only if $\text{Re}\bar{\xi}_{\bar{f}} \neq \text{Re}\xi_f$. There is a handful of neutral B decays, e.g., $B_d^0 \rightarrow \bar{D}^{0(*)} K_S$, $D^{(*)\pm} \pi^\mp$, and $\bar{D}^{(*)0} \pi^0$, which satisfy the conditions $|\bar{\xi}_{\bar{f}}| = |\xi_f|$ and $\text{Re}\bar{\xi}_{\bar{f}} \neq \text{Re}\xi_f$. Taking $B_d^0/\bar{B}_d^0 \rightarrow D^+ \pi^-$ for example, an isospin analysis shows that

$$\begin{aligned}\xi_{D^+ \pi^-} &= \frac{V_{cb} V_{ud}^*}{V_{cd} V_{ub}^*} \cdot \frac{\bar{a}_{3/2} e^{i\delta_{3/2}} + \sqrt{2} \bar{a}_{1/2} e^{i\delta_{1/2}}}{a_{3/2} e^{i\delta_{3/2}} - \sqrt{2} a_{1/2} e^{i\delta_{1/2}}}, \\ \bar{\xi}_{D^- \pi^+} &= \frac{V_{ud} V_{cb}^*}{V_{ub} V_{cd}^*} \cdot \frac{\bar{a}_{3/2} e^{i\delta_{3/2}} + \sqrt{2} \bar{a}_{1/2} e^{i\delta_{1/2}}}{a_{3/2} e^{i\delta_{3/2}} - \sqrt{2} a_{1/2} e^{i\delta_{1/2}}}, \quad (38)\end{aligned}$$

where V_{ij} ($i = u, c, t; j = d, s, b$) are the Cabibbo-Kobayashi-Maskawa matrix elements; $a_{3/2}$ ($\bar{a}_{3/2}$) and $a_{1/2}$ ($\bar{a}_{1/2}$) are the isospin amplitudes for a B_d^0 (\bar{B}_d^0) decaying into $D^+ \pi^-$ ($D^- \pi^+$) and its CPT -conjugate process [15], and $\delta_{3/2}$ and $\delta_{1/2}$ are the corresponding strong phases. Obviously $|\bar{\xi}_{D^- \pi^+}| = |\xi_{D^+ \pi^-}|$ holds, but $\bar{\xi}_{D^- \pi^+} \neq \xi_{D^+ \pi^-}^*$ because of $\delta_{3/2} \neq \delta_{1/2}$. As a result, measurements of \mathcal{A}_-^S in such decay modes may serve as a good test of CPT symmetry in the B -meson decays.

From Eqs.(35) and (36) we see that \mathcal{A} and \mathcal{A}_\pm^S are also modified in the presence of nonvanishing $\tilde{\mathcal{W}}_-$ or \mathcal{S} . However, it is difficult to extract any information on CPT violation from them. In principle, measurements of the time-dependent asymmetries $\mathcal{A}(t)$ and $\mathcal{A}_\pm^A(\Delta t)$ are possible to probe \mathcal{S} with less ambiguity. For example, nonvanishing CPT violation can be extracted from

$$\mathcal{A}\left(\frac{(2n+1)\pi}{\Delta m}\right) = \mathcal{A}_-^A\left(\frac{(2n+1)\pi}{\Delta m}\right) = \frac{2\tilde{\mathcal{W}}_-}{1 - \tilde{\mathcal{U}} + 2\tilde{\mathcal{W}}_+}, \quad (39)$$

and

$$\mathcal{A}_+^A\left(\frac{n\pi}{\Delta m} - \frac{\phi_{x_d}}{\Delta m}\right) = \frac{\tilde{\mathcal{W}}_- \left[\frac{4-x_d^2}{4+x_d^2} \sqrt{1+x_d^2} - (-1)^n \right]}{\sqrt{1+x_d^2} + (-1)^n \tilde{\mathcal{U}} + \tilde{\mathcal{W}}_+ \left[\frac{4-x_d^2}{4+x_d^2} \sqrt{1+x_d^2} - (-1)^n \right]} \quad (40)$$

where $n = 0, \pm 1, \pm 2$, and so on.

IV. Conclusion

In order to probe the sources of CP and CPT violation in neutral B -meson decays, we have explored various possible measurements at e^+e^- B factories and hadron machines. It is shown that CPT -violating effects can in principle be distinguished from direct and indirect CP -violating effects, by measuring the time development of CP asymmetries $\mathcal{A}(t)$ and \mathcal{A}_\pm^A . A clean signal of CPT violation may appear in the time-integrated asymmetries \mathcal{A}_-^S for some non- CP -eigenstate decay modes, e.g., $B_d^0/\bar{B}_d^0 \rightarrow D^\pm \pi^\mp$ and $\bar{D}^{(*)0} K_S$. In contrast, measuring \mathcal{A}_-^S in the CP -eigenstate decays such as $B_d^0/\bar{B}_d^0 \rightarrow \psi K_S$ and $\pi^+\pi^-$ cannot provide a good limit on CPT violation, since direct CP violation may compete with or dominate over CPT violation in them.

In keeping with the current interest in tests of discrete symmetries and conservation laws in the K -meson system [16], the parallel approaches are worth pursuing, especially on investigating CP violation and checking CPT invariance, for weak decays of B mesons. With the development in building high-luminosity B factories [9,11,12], it is possible to observe CP (or T) violation in neutral B decays in the near future. The fine effects of CPT violation, if they are present, may be probed in the second-round experiments at a B factory.

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